Rational Investor Behaviour and Market Mispricing
– The Resale Option Pricing Effect

by Henrik Anderson* and Kenth Skogsvik*

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The fundamental value of a company share is commonly calculated as the present value of future expected dividends, typically modelled as some function of the firm's future accounting earnings and/or book values. However, when substantial magnitudes of trades in the stock market are made by "uninformed" investors, the observed price might only be a noisy reflection of this fundamental value. For an informed investor, this potentially creates a value-enhancing opportunity. In principle, the investor can either wait for the dividends to be realised, or sell the stock. It can be rational to sell when there is a mispricing, such that the share price is higher than the present value of future expected dividends, but the intriguing question is: How much higher than the fundamental value should the price be to sell? We formalize this question as a so called stopping problem, and show that substantial mispricing typically is required before any trading should take place. Hence, market mispricing cannot really be expected to "immediately" correct itself, but can actually be present for long periods of time. Our modelling sheds new light on stock market bubble phenomena, as well as provides a tool for fund managers to rationally determine trading strategies in their portfolio management.

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* Henrik.Andersson@hhs.se; Kenth.Skogsvik@hhs.se, Stockholm School of Economics, Box 6501, S – 113 83 Stockholm, Sweden. For discussions and valuable comments on an earlier version of this paper, the authors are much indebted to Sidney Howell, Peter Jennergren and Martin Walker.

1. Introduction

The efficient market hypothesis (EMH), i.e. the idea that financial assets are "correctly" priced, has its proponents and critics. However, typically not even its most convinced proponents claim that financial assets always are correctly priced. For example, the market crash of October 1987, or the IT-bubble around the turn of the millennium, are indeed difficult to explain based on fundamental values only. On the other hand, critics of market efficiency typically admit that stock market prices must at least have something to do with the accounting based fundamentals of companies. There are hypes, but sooner or later the underlying fundamentals should matter.

Arguably stock market prices can be viewed as noisy reflections of the present value of future expected dividends. In principle, this implies that an investor who can assess this (intrinsical) value of expected dividends has an information advantage as compared to investors who only observe and accept the market price. In particular, an informed investor has the opportunity to sell the share when it is overvalued – i.e. in addition to the fundamental value of the share, he/she also possesses a resale option. An equity valuation problem of this kind can be viewed as an optimal stopping problem, which can be solved given the stochastics of the stock price. We propose a continuous time process for the share price dynamics, and use this process to estimate the value of this resale option.

Perhaps the most widespread model of 'irrational' pricing stems from Summers (1986). In the specification by Summers, (the logarithm of) the stock price is the sum of an underlying fundamental value and random noise. The fundamental value is assumed to follow a random walk, and the noise component is modelled as an autoregressive AR(1) process. Summers' model was proposed as a simple alternative to the (naïve) idea that share prices always are 'correct', and with two sources of randomness the model implies that the volatility of the share price will be higher than the volatility of the fundamental value alone.

The belief that stock prices always are 'correct' had previously been challenged by a number of researchers. Shiller (1981) showed empirically that the stock price volatility was

at least 5 times higher than what could be explained by unexpected changes in company dividends. On the other hand, LeRoy and Porter (1981) argued that share price volatility should be lower than the volatility of company earnings. The present value of dividends is a weighted average of possible future dividends and should therefore in general be less volatile. Similarly, expected earnings should show less variation than realized earnings for the distant future. However, Campbell and Shiller (1987) provided further evidence of excess volatility in the stock market, based on co-integration tests of dividends and stock prices. More lately, there is evidence that the association between fundamental values and share prices has become distinctively weaker over time. Curtis (2012) not only confirmed that the excess volatility in market prices appears to be persistent, but also showed that the co-movements between accounting based fundamental values and market prices have become lower for the US stock market after the mid-nineties. This potentially indicates that market mispricing might be even more pronounced nowadays.

As an illustration of our reasoning, consider the historical development of the share price of Google. Google has shown a stable earnings growth over the last 7 years (from October 2008 to September 2015). No dividends have been distributed during this period, and the equity book value has grown at a steady pace over time. Multiplying the equity book value with an average market-to-book ratio of 3.92 during these years, generates a naïve fundamental valuation represented by the stable upwards sloping line in Figure 1 below. Yet, the stock price of Google has been far more erratic. The return volatility of Google is 22.2 % over this period, whereas it is only 4.5 % for our simple fundamental value. Clearly, applying a constant market-to-book multiple constitutes a crude measurement of the equity fundamental value, but given the stable earnings growth it might not be too unrealistic.[1]

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[1] In fact Google's earnings growth was stable also before October 2008, but as it arguably took some time for the market expectations for a new business model like Google's to settle down, we have excluded these observations in the figure.
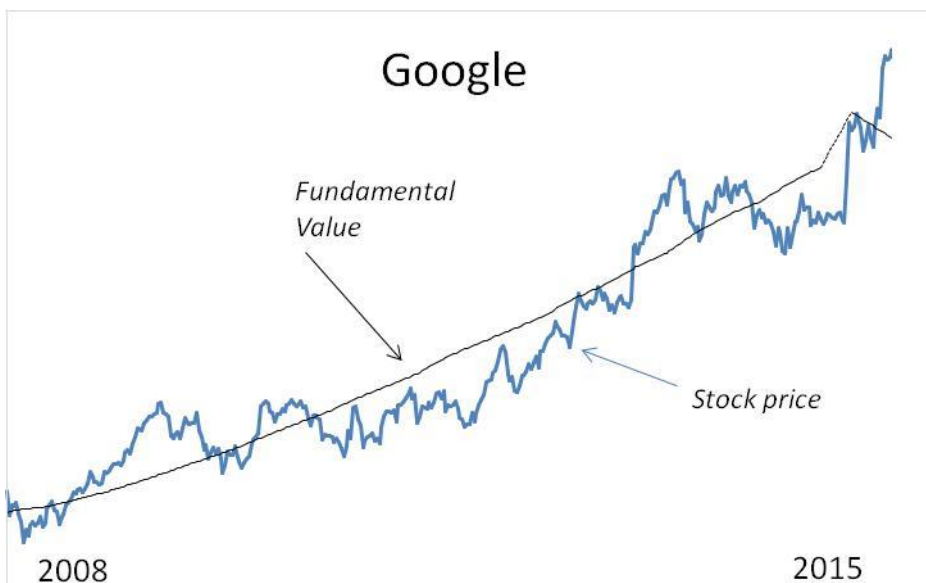
Figure 1. Google's stock price and estimated fundamental value from
October 2008 to September 2015.

In Harrison and Kreps (1978) it was claimed that someone who is prepared to pay more for a share that can be sold, as compared to one that must be held "forever", displays speculative behaviour. The authors showed this by means of a simple discrete time model, where the ability to sell the share at a higher price than the value of future dividends creates extra value for someone that is informed about this discrepancy. Non-informed traders, often referred to as 'noise-traders', cause the stock market price to be a noisy reflection of the underlying fundamental value. Generally speaking, noise-trading include chartists, "positive-feedback" traders and others relying on pseudo signals. Robot trading and other automated execution algorithms can potentially also be included, as well as investors who simply are ignorant.

In a stock market characterised by fierce investor competition, fundamental investors are expected to sell (or short) a share as soon as there is a profit to be made. If not, someone else is expected to seize the opportunity to make money out of the mispricing, and the observed mispricing will be gone. Given this idea about how erroneous share prices are

corrected, the mispricing should be neither large nor long-lived. Or, in the words of Shleifer and Vishny:

'The efficient market approach is based on the assumption that most investors, like the economists, see the available arbitrage opportunities and take them.' (Shleifer and Vishny, 1997; p. 52)

Shleifer and Vishny argued that there are limits to the ability of arbitrageurs to trade away mispricing caused by noise-traders. Margin requirements, endowments, organisational aspects and contract differences can all contribute to arbitrage ineffectiveness. Pontiff (2006) also makes the point that in the presence of mispricing, the market portfolio is no longer optimal and that therefore also idiosyncratic risk can be of importance.

In the aftermath of the IT-bubble and the notorious "accounting scandals" in the beginning of this millennium, a strand of literature emerged that looked at shares that have become overvalued. Jensen (2005) provides some theoretical underpinnings by describing the agency problems of overvalued stocks. Management and board members become trapped in a game trying to sustain an exaggerated price as many targets and compensation schemes are tied to the share price. On the empirical side, Moeller, Schlingemann and Stulz (2005) found that companies with high market to book values are more prone to make value destroying acquisitions, and Badertscher (2011) found that earnings management is more common the longer a share is overvalued.

In spite of the above findings, the efficient market hypothesis still appears to be the prominent benchmark for our understanding of financial asset prices. If fundamental investors are dominating the market trading, we should expect market prices to be efficient up to the point where the marginal cost of obtaining additional information equals the marginal benefit from trading on that information (Jensen, 1978; Grossman and Stiglitz, 1980; Fama, 1991). In contrast to a market where efficiency is immediate, Bloomfield (2002) calls this process of gradual and incomplete information incorporation into security prices the "incomplete revelation hypothesis". Generally then, the degree of market efficiency becomes a battle between the relative strength of fundamental investors and

noise-traders. Grossman and Stiglitz (1980), De Long, Shleifer, Summers and Waldmann (1990), and Hong and Stein (1999), are some examples where deviations from the immediate efficient market is explained by differences between investor clienteles.

Related to our paper is the theoretical equilibrium model of Scheinkman and Xiong (2003), where two investor clienteles have heterogeneous beliefs about the dividend growth rate of shares. Scheinkman and Xiong argued that short sale constraints imply that only the view of one group of investors can be reflected in the stock price at each point in time. They showed that as the market power shifts between the two clienteles a pricing bubble can emerge, subsequently accompanied by extensive trading and excess price volatility. The authors labelled the difference between a share's market price and its underlying fundamental value as the *resale option value*, stemming from Harrison and Kreps (1978) and the notion of shareholders being able to resell shares at a higher price.

The predictions made by the model in Scheinkman and Xiong (2003) was empirically tested in Chen, Lung and Wang (2009), finding that price bubbles are indeed accompanied by excess volatility and intense trading. Dumas, Kurshev and Uppal (2009) generalised the model by allowing for risk adverse investors and short selling, as well as clarifying the importance of the distinction between informed investors and noise traders.

Given that mispricing actually occurs and the resale option exists - what is the value of this insight? We utilize dynamic programming and provide a solution to the problem of when an investor should sell a mispriced share. We then assume that the market price process is exogenously determined. Even though our pricing model is no equilibrium model, it nevertheless has equilibrium consequences. Our modelling indicates that stock market mispricing potentially can be large and that the time before the optimal exercise price is reached can be substantial. Bubbles can thus remain for long periods of time. The existence of an optimal selling price is dependent on the presence of mispricing; i.e. that the stock price differs from the underlying fundamental value (i.e. the present value of expected future dividends). An investor who knows the present value of future expected dividends

therefore has an opportunity to explore this information advantage. In principle, it is the value of this information advantage that we model.

A simplified example can serve as an illustration of our modelling logic. Suppose the stock price is $10, a value coinciding with the present value of future expected dividends. All of a sudden –without any change of the underlying fundamental value – the stock price increases to $11. Fundamental investors holding the share can now sell it, an action that might drive the price back (close) to $10. Selling immediately may be the best strategy if the mispricing is small and market corrections are fast. However, if there is an expectation of more persistent mispricing in the stock market, the sell decision of fundamental investors might change. If that is the case, it might be better for the fundamental investors to wait, as there is a possibility that the occurred mispricing actually might increase.

Exercising the resale option and selling a share is an irreversible act. Selling a mispriced share is similar to the problem of deciding when to optimally make an irreversible investment. The standard idea in capital budgeting problems is to invest as soon as the present value of future expected cash flows is higher than the investment outlay. However, as emphasised in the real options literature, it is not necessarily optimal to invest immediately (cf. McDonald and Siegel 1986; Dixit and Pindyck, 1994). It may be better to wait and observe how the output prices develop before committing to an irreversible investment. In the same way, it is not necessarily optimal to sell a share as soon as there is a positive difference between the stock price and the fundamental value. The fundamental value constitutes a lower limit of the value of the share, and it might be advantageous to wait and allow the observed mispricing to develop further.

The paper continues as follows. In the next section we describe the stock price process and our characterisation of market mispricing. In section 3, the optimal stopping problem of when to exercise the resale option is presented and solved. Section 4 includes a numerical example and describes potential economic consequences of our valuation modelling. Section 5 includes some concluding remarks. Some derivations are included in appendices; the solution to the stock price process is showed in Appendix A, and some properties of the

mispricing are derived in Appendix B. Appendix C includes definitions of the more important variables and parameters that are used in the paper.

2. The stock price process

The first step is to model a stochastic process for the behaviour of stock prices in the presence of noise traders. The present value of expected dividends, PVED, will be labelled the fundamental value of the stock, and denoted $V(t)$. Of course, it does not have to be calculated through a dividend discount model. Alternative specifications includes more accounting based models such as the Residual Income Valuation model of Ohlson (1995), or the Abnormal Earnings Growth model of Ohlson and Juettner-Nauroth (2005).[2] The fundamental value $V(t)$ is assumed to follow a geometric Brownian motion, i.e.:

$$dV = (\mu - \delta)Vdt + \sigma_V VdW_V. \tag{1}$$

The drift in the fundamental value is specified as the difference between the risk-adjusted required return for holding the stock μ and the dividend yield δ, reflecting that returns can either be in the form of dividends or stock price changes. It is not a smooth process, however, and thereby the diffusion parameter $\sigma_V . dW_V$ is the increment of a Wiener process, representing changes in expectations about future dividends. In accordance with (1), there is a dividend process linked to the fundamental value, where the dividend is:

$$dD = \delta Vdt, \tag{2}$$

during a short time interval dt.

So far, everything has been according to a standard Black-Scholes setting. Now, assume that the stock price is a noisy reflection of the fundamental value so that there is a

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[2] In model comparisons, the residual income valuation model commonly fairs well, cf. Francis et al (2000) and Jorgensen et al (2011).

difference between the fundamental value and the observed stock price. The price is modelled as mean-reverting according to the following stochastic process:

$$dS = \left[\theta + \eta(\ln V - \ln S)\right]Sdt + \sigma_S SdW_S. \tag{3}$$

The specification in (3) ensures that the stock price is log-normal, as the logarithm of the stock price follows a so called Ornstein-Uhlenbeck process and, importantly, there exists an analytic solution to this stock price dynamics (as shown in Appendix A).

If the (log) fundamental value is higher than the (log) observed price, there is a drift upwards in the price, but if the price is too high there is a drift downwards. η represents the speed of reversion, and θ is a correction term that will ensure that the observed price is asymptotically equal to the fundamental value when there is no new information. Mispricing is therefore in expectation corrected.[3] σ_S is the 'irrational' stock price volatility caused by noise traders. The Wiener processes in (1) and (3) are possibly correlated with $Corr[dW_V, dW_S] = \rho$. If the reversion dynamics is sufficiently slow and the irrational volatility is sufficiently high, the difference between the fundamental value and the observed stock price can be large.

In the numerical example illustrated in Figure 2 below, the dividend yield is set to $\delta = 0$, and the geometric Brownian motion of the fundamental value has a drift rate equal to the required return $\mu = 8\%$, with a standard deviation of $\sigma_V = 20\%$. The speed of reversion towards the fundamental value is $\eta = 0.5$. The interpretation of η is that without any new information arriving, $e^{-\eta \cdot 1} = 60\%$ of the mispricing remains after 1 year. The 'irrational volatility' is $\sigma_S = 20\%$ and for the realisation shown in Figure 2, the combined effect of σ_V and σ_S translates into a return standard deviation of 36 % per annum for the stock price. Finally, the two processes are assumed to be uncorrelated, i.e. $\rho = 0$.

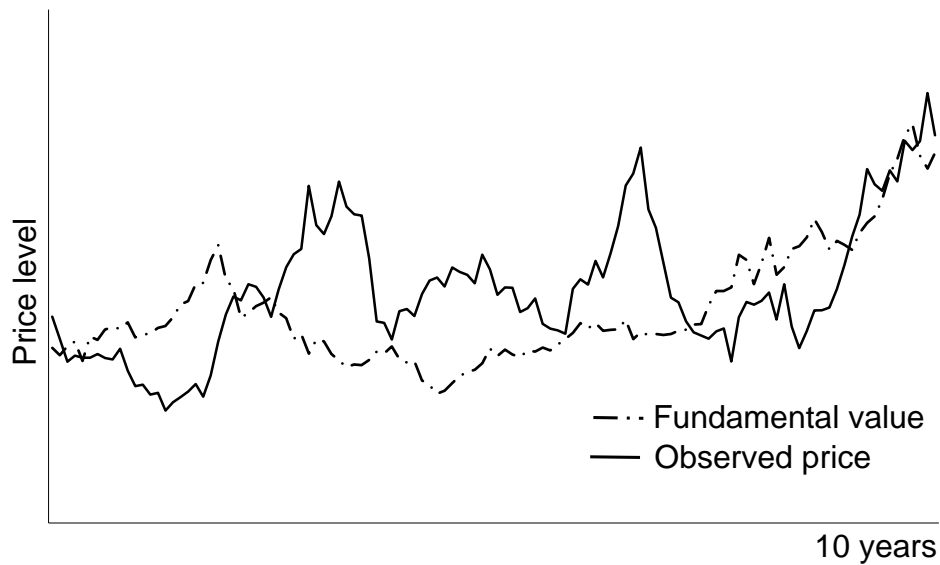
[3] See footnote 1 for the definition of θ.

Figure 2. One realisation of the price process from equations (1) and (3).

Some stylised assumptions being incorporated into the model are as follows:

1) Both *V* and *S* are lognormally distributed, disallowing negative fundamental values and stock prices.

2) The observed stock price converges towards the fundamental value if no new information arrives.

3) The main driver of the stock price is the fundamental value, but this is not fully reflected in the observed price as this also contains a random component (the 'mispricing').Hence the stock price volatility is higher than the volatility of the fundamental value (in line with Shiller, 1981).

4) The stock price contains both a permanent and a temporary component (cf. Cochrane, 1988; Fama and French, 1988; Lo and MacKinlay, 1988.) For a random walk the variance increases linearly over time, whereas for a mean reversion process the variance is asymptotically constant. Our modelling of the stock price incorporates both as depicted in Figure 3 and Appendix A, equation (A.10). [4]

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[4] An equivalent claim is to say that the variance ratio is less than one.
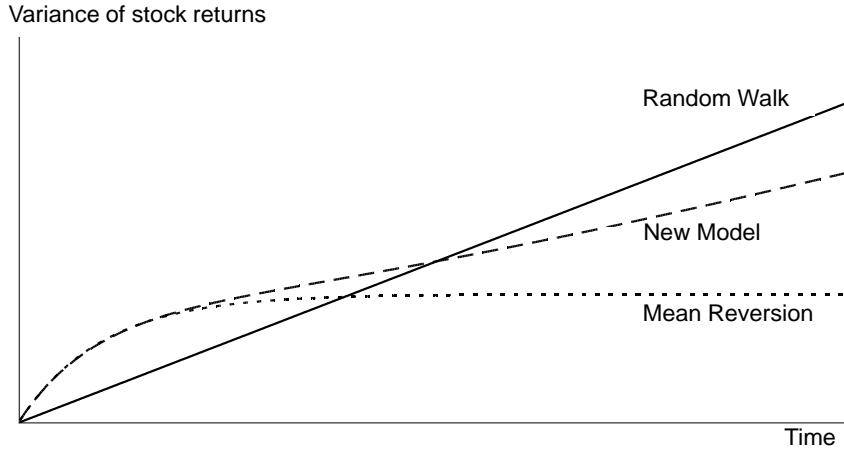
Figure 3. Asymptotic variance behaviour in our resale option price model (from
equation (A.10) in appendix A).

Although our suggested pricing model is more elaborate, it can in discrete time be said to

converge to the simple model suggested in Summers (1986), with a random walk and an

additional AR(1) error term. Contrary to the equilibrium models in Scheinkman and Xiong

(2003), we have no limitations on short selling. The ability to short sell a stock should

reduce the magnitude and length of the mispricing. In our model this translates into a

smaller volatility σ_S and a higher speed of reversion $\eta,$ as compared to a situation when no

short selling is allowed. If short selling causes the volatility of mispricing to be 0 and hence

the speed of price reversion to be instantaneous, the price process collapses into the

standard Black-Scholes price process, with no optimal exercise price.[5]

3. The resale option

We have described the stock price process in section 2 above, and the next step is to solve

the optimal stopping problem of when to sell a share. The resale option denoted $G(S,V,t)$

on the mispriced share is not traded. It is the private value of knowing the fundamental

value V while observing the stock price S. In order to determine its value, and thereby the

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[5] The correction term $\theta = \frac{1}{2}\sigma_S^2 + \mu - \delta - \frac{1}{2}\sigma_V^2,$ ensures that when $\sigma_S \to 0$ and $\eta \to \infty,$ equation (A.11)
becomes the solution to the Geometric Brownian Motion in (1).

optimal selling point, we cast it as an optimal stopping problem and utilize dynamic programming for the solution.

$$G(S,V,t_0) = \max_{S^*,V^*} e^{-\mu(t^*-t)} E_{t_0}\left[\int_{t_0}^{t^*} \delta V dt + G(S^*,V^*)\right] \tag{4}$$

subject to:

$$G(S^*,V^*) = S^* - V^*$$

$$dV = (\mu - \delta)Vdt + \sigma_V V dW_V$$

$$dS = \left[\theta + \eta(\ln V - \ln S)\right]Sdt + \sigma_S S dW_S$$

The first term in the within the expectation bracket merely says that a dividend δV is received for every unit of time the stock is owned. All action is in the second term. We seek to determine an optimality condition, denoted by *, when the stock should be sold for the observed price S^* and hence giving up the future dividends worth V^*. The proceeds from such an exchange is obviously $S^* - V^*$.

As long as the value of the resale option G is higher than the current difference between the observed price and the fundamental value $(S - V)$, the resale option is kept alive - otherwise it should be exercised. The choice of exercising immediately or keeping the option alive for one more period leads to the Bellman principle of optimality, i.e. that an optimal solution must be optimal at all times. If it is not optimal to exercise immediately, we can hold the option one more period and then solve the problem at that time. The logic is the same both for continuous and discrete time and a heuristic derivation of the stochastic Bellman equation is to cast the problem in discrete time. For the small time-increment Δt we can then write the equivalent dynamic programming problem as,

$$G(S,V,t_0) = \max E_{t_0}\left[S - V \; ; \; \delta V \Delta t + \frac{1}{1 + \mu \Delta t} G(S,V,t_0 + \Delta t)\right], \tag{5}$$

under the same conditions as before. The value of keeping the option alive one more period - the continuation value - then incorporates the possibility of exercising the option in all possible future periods. This is the actually key to the solution. Concentrating on the second part of (5), i.e. the continuation value, we can write

$$G(S,V,t_0) = \delta V \Delta t + \frac{1}{1 + \mu \Delta t} E_{t_0} \left[G(S,V,t_0 + \Delta t) \right]. \tag{6}$$

Multiplying by $(1 + \mu \Delta t)$ and rearranging terms gives:

$$\mu G(S,V,t_0) \Delta t = (1 + \mu \Delta t) \delta V \Delta t + E_{t_0} \left[G(S,V,t_0 + \Delta t) - G(S,V,t_0) \right]. \tag{7}$$

Letting $\Delta t \to dt$, and therefore $(\Delta t)^2 \to 0$, gives the differential equation:

$$\mu G(S,V,t) dt = \delta V dt + E(dG). \tag{8}$$

This is the Bellman equation for the continuation region where the resale option is not exercised. The logic of equation (8) is to say that the required return for holding the resale option (the LHS) can come in the form of dividends and/or an appreciation in the value of the resale option (the RHS). Of course, there is nothing unique for the resale option in this logic, equation (8) is generic. The specificity of a contract is determined by the boundary condition. In this case, the boundary condition is to break when the value of keeping the option alive no longer exceeds the value from an immediate exercise. In other words, it is an optimal stopping problem.[6]

In order to demonstrate the dynamic programming technique, let us use the analogy of a simple one-period financial contract *X* which has an expected pay-off next year of $11 and a market price today of $10. Another way to express this is that the current price reflects an

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[6] More details of dynamic programming can be found in Dixit and Pindyck (1994), 'Investment under Uncertainty' (1994). Our equation (8) is the equivalent of equation (8) in Dixit and Pindyck (1994), chapter 4.

expected profit of $1, or $E(\Delta X) = \$1$. The question is then whether this is enough for investors to hold on to the contract. If investors require the return μ for holding this risky contract, we can write $\mu X = E(\Delta X)$ as the equilibrium condition for the price of the contract. The simple interpretation is that if $\mu = 10\%$, the *required* income of $1 matches the *expected* income of $1. The important point is that this equilibrium condition together with the expected payment of $11 (the boundary condition) is enough to determine the price of $10 today. Of course, there is nothing special about a time interval of one year, and more generally we can write $\mu X dt = E(dX)$ as the equilibrium condition. This is the same logic as for determining the value of the resale option. Instead of the contract *X*, we have then the resale option *G* and the dividend attached to it.

Generally speaking, dynamic programming and option theory give the same solution, but there are a few modelling differences. In option theory, one creates a portfolio that is instantaneously risk-free and thereby forces any percentage change in value to equal the risk-free rate. In dynamic programming, a relative change in value between two points in time is equal to the risk-adjusted required return μ, but there is no guidance of how this risk-adjusted required return should be determined. Dynamic programming is based on a the concept of equilibrium, and not the binding requirement of arbitrage. On the other hand, option pricing requires an assumption about a market price of some risk when there is more than one source of randomness.[7] This is not required when dynamic programming is used. For this problem, however, option theory is not applicable as the market is not arbitrage-free in the presence of mispricing. Even so, it is possible to cast the resale option as an optimal stopping problem and solving it through the Bellman equation.

In equation (1), μ is the required return for determining the present value of future expected dividends. In (8), μ is the required return being appropriate for the resale option. As the resale option implies that an investor holds the share, there is no reason to have a

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[7] For example, if volatility is stochastic a market price of volatility risk has to be assumed.

required return different from μ, as long as mispricing is an idiosyncratic risk. This motivates the specification $\mu G(S,V)dt$.[8]

Armed with the processes in (1) and (3) and Ito calculus, we can to expand dG in equation (8) to get:

$$dG = \left\{ G_t + (\mu - \delta)VG_V + [\theta + \eta(\ln V - \ln S)]SG_S + \tfrac{1}{2}\sigma_V^2 V^2 G_{VV} + \tfrac{1}{2}\sigma_S^2 S^2 G_{SS} + \rho\sigma_V\sigma_S VSG_{VS} \right\} dt \quad (9)$$
$$+ \sigma_V VG_V dW_V + \sigma_S SG_S dW_S.$$

Noting that $\sigma_V VG_V dW_V$ is the shorthand for the integral $\int_0^t \sigma_V V(\tau)G_V(S,V,\tau)dW_V(\tau)$, which

in expectation is 0 (cf. Björk, 1998; p. 42), we get

$$E(dG) = \left\{ G_t + (\mu - \delta)VG_V + [\theta + \eta(\ln V - \ln S)]SG_S + \tfrac{1}{2}\sigma_V^2 V^2 G_{VV} + \tfrac{1}{2}\sigma_S^2 S^2 G_{SS} + \rho\sigma_V\sigma_S VSG_{VS} \right\} dt.$$

Substituting this expression into (8), gives the following partial differential equation to be satisfied for each dt:

$$\begin{aligned} & G_t + (\mu - \delta)VG_V + \theta SG_S + [\theta + \eta(\ln V - \ln S)]SG_S + \\ & + \tfrac{1}{2}\sigma_V^2 V^2 G_{VV} + \tfrac{1}{2}\sigma_S^2 S^2 G_{SS} + \rho\sigma_V\sigma_S VSG_{VS} - \mu G = -\delta V. \end{aligned} \quad (10)$$

The solution to the above equation can be split into the sum of two parts: The general solution to the homogeneous differential equation, where the right hand side of (10) is 0, and one particular solution to the full differential equation, so that $G = G_{\text{Homogenous}} + G_{\text{Particular}}$. In this case, one particular solution with an obvious economic interpretation is easy to spot, $G_{\text{Particular}}(S,V,t) = V$. Holding a stock means receiving future dividends having the present

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[8] Choosing a risk adjusted return different from μ is problematic as the particular solution to equation (10) is then no longer the fundamental value V. The solution to the homogenous part of the differential equation can in such a case only vaguely be interpreted as the value of the resale option. The fallacy here is that the solution to the resale option and the fundamental value cannot be added to obtain the total value of the stock. Another problem of choosing a risk adjusted return different from μ, is that there will be no analytical solution.

value *V*. In principle, dividends are the only cash flows attached to the ownership of a share. Any solution to the homogenous part of (10) is then a result of mispricing.

There are three state variables in the differential equation, i.e. $\{S,V,t\}$. These can be reduced to two for the resale option. The option to sell the stock and 'cash in' on the mispricing has no maturity date; the problem looks exactly the same next year as it does today (although the numerical parameters and state variables might differ).[9] Therefore, $G_t = 0$ and the first term in (10) disappears. We are left with:

$$(\mu - \delta)VG_V + \left[\theta + \eta(\ln V - \ln S)\right]SG_S + \tfrac{1}{2}\sigma_V^2 V^2 G_{VV} + \tfrac{1}{2}\sigma_S^2 S^2 G_{SS} + \rho\sigma_V\sigma_S VSG_{VS} - \mu G = 0$$

(11)

Generally, (11) must be solved numerically. However, there is one special case when it is possible to obtain an analytical solution – the case when no dividends are received before the optimal exercise condition is met.

3.1. The special case of no dividends

Not allowing any dividends to be paid out (i.e. $\delta = 0$) before the optimality condition is met is a severe restriction, but the properties of the solution under no dividends will carry through also to a numerical solution including dividends. Currently around 120 of the S&P500 companies are not paying dividends. Among companies that have never paid dividends we find Google, Yahoo, eBay, Amazon and Berkshire Hathaway. Companies with a stable history of positive dividends are the typically easiest to analyse and mispricing can be expected to be smaller. Bubbles are more prone to occur in so called growth stocks paying no or very small dividends. The no dividends case is in this sense the worst case from a market efficiency point of view.

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[9] The simplest example of a financial contract without a time dimension is a perpetuity with one cash flow in each period and therefore a present value PV = C/r. The value will stay the same as long as the discount rate does not change.

When elaborating on the question of when to optimally sell a mispriced stock, the most natural addition is perhaps to continue with a rhetoric question: Should it be when it is overpriced with 5 %, 50 % or even 500 %? This seemingly innocent question actually conceals an extremely important observation: It is only the relation between the observed stock price S and the fundamental value V that matters, the relative mispricing, not the values in themselves. The solution is independent of how many shares the company value is split into. We therefore set the mispricing ratio

$$R(S,V) = \frac{S(V)}{V} \tag{12}$$

as our dependent variable. However, even though optimal exercise is only dependent on this ratio, the value of the option is still dependent on the size of the position. Formally, we make the substitution,

$$G(S,V) = V \cdot H(S/V) = V \cdot H(R), \tag{13}$$

and solve the differential equation (13) for H. This means that V will drop from the differential equation, making it a function of R only. Differentiation now gives:

$$G_S = \frac{\partial G}{\partial S} = V \cdot \frac{\partial H}{\partial R} \cdot \frac{\partial R}{\partial S} = V \cdot \frac{\partial H}{\partial R} \cdot \frac{1}{V} = \frac{\partial H}{\partial R} = H_R \tag{14}$$

$$G_{SS} = \frac{\partial^2 H}{\partial R^2} \cdot \frac{\partial R}{\partial S} = \frac{\partial^2 H}{\partial R^2} \cdot \frac{1}{V} = \frac{1}{V} \cdot H_{RR} \tag{15}$$

$$G_V = \frac{\partial G}{\partial V} = H + V \cdot \frac{\partial H}{\partial R} \cdot \frac{\partial R}{\partial V} = H + V \cdot \frac{\partial H}{\partial R} \cdot \left(-\frac{S}{V^2}\right) = H - R \cdot H_R \tag{16}$$

$$\begin{aligned} G_{VV} &= \frac{\partial^2 G}{\partial V^2} = \frac{\partial H}{\partial R} \cdot \frac{\partial R}{\partial V} - \frac{\partial R}{\partial V} \cdot \frac{\partial H}{\partial R} - R \cdot \frac{\partial^2 H}{\partial R^2} \cdot \frac{\partial R}{\partial V} \\ &= -R \cdot \frac{\partial^2 H}{\partial R^2} \cdot \left(-\frac{S}{V^2}\right) = \frac{R^2}{V} \cdot H_{RR} \end{aligned} \tag{17}$$

$$G_{SV} = \frac{\partial^2 H}{\partial R^2} \cdot \frac{\partial R}{\partial V} = -\frac{S}{V^2} \cdot \frac{\partial^2 H}{\partial R^2} = -\frac{R}{V} \cdot H_{RR} \tag{18}$$

Substituting (13) to (18) into (11) gives with $\delta = 0$, the differential equation:

$$\mu V(H - RH_R) + \left[\theta + \eta(\ln V - \ln S)\right]SH_R + \tfrac{1}{2}\sigma_V^2 VR^2 H_{RR} +$$
$$+ \tfrac{1}{2}\sigma_S^2 S^2 \frac{1}{V} H_{RR} - \rho\sigma_V\sigma_S SRH_{RR} - \mu VH = 0. \tag{19}$$

Expression (19) can be rewritten by dividing through by V and substituting $R = S/V$:

$$\mu(H - RH_R) + (\theta - \eta\ln R)RH_R + (\tfrac{1}{2}\sigma_V^2 + \tfrac{1}{2}\sigma_S^2 - \rho\sigma_V\sigma_S)R^2 H_{RR} - \mu H = 0 \tag{20}$$

Collecting terms and substituting $a = \tfrac{1}{2}(\sigma_V^2 + \sigma_S^2 - 2\rho\sigma_V\sigma_S)$ gives:

$$aR^2 H_{RR} + (\theta - \mu - \eta\ln R)RH_R = 0 \tag{21}$$

(21) can now be developed further by substituting:

$$b = \theta - \mu = \tfrac{1}{2}\sigma_S^2 + \mu - \delta - \tfrac{1}{2}\sigma_V^2 - \mu = \tfrac{1}{2}(\sigma_S^2 - \sigma_V^2) - \delta \tag{22}$$

where for this case of no dividends, we can write the differential equation as:

$$aR^2 H_{RR} + \left[b - \eta\ln R\right]RH_R = 0. \tag{23}$$

Interestingly the required return μ has now vanished from the differential equation. It cancelled out in the expression for the parameter b in (22). The explanation for this is that what now matters is the ratio between the share price and its fundamental value. The drift of the fundamental value over time will not affect the solution as time is no longer a state variable in the differential equation.

Wolfram Alpha (*Mathematica*) provides the general solution to the differential equation in (23) as

$$H(R) = A_1 \sqrt{\frac{\pi a}{2\eta}} e^{-\frac{(a+b)^2}{2a\eta}} \, erfi\left(\frac{a + \eta \ln R + b}{\sqrt{2a\eta}}\right) + A_2. \tag{24}$$

In (24), e*rfi* is the 'imaginary error function', defined as

$$erfi\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{t^2} dt. \tag{25}$$

The imaginary error function has a series representation,

$$erfi\,(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{x^{2k+1}}{k!(2k+1)}, \tag{26}$$

which comes in handy when the problem is solved in excel. We will also need the derivative $H_R(R)$. Differentiation of the imaginary error function gives:

$$\frac{\delta}{\delta x} erfi\,(x) = \frac{\delta}{\delta x}\left(\frac{2}{\sqrt{\pi}} \int_0^x e^{t^2} dt\right) = \frac{2}{\sqrt{\pi}} e^{x^2}, \tag{27}$$

and thus

$$H_R(R) = A_1 \sqrt{\frac{\pi a}{2\eta}} e^{-\frac{(a+b)^2}{2a\eta}} \cdot \frac{2}{\sqrt{\pi}} e^{\left(\frac{a+\eta \ln R+b}{\sqrt{2a\eta}}\right)^2} \cdot \frac{\eta}{R\sqrt{2a\eta}}, $$

which can be simplified to:

$$H_R(R) = A_1 e^{-\frac{(a+b)^2}{2a\eta}} \cdot e^{\left(\frac{a+\eta \ln R+b}{\sqrt{2a\eta}}\right)^2} \cdot \frac{1}{R}. \tag{28}$$

Having arrived at the solution to the differential equation, we use the standard arguments of an absorbing barrier, value matching and smooth pasting to determine the constants in (24) and solve this stopping problem.

Absorbing barrier

The constant A_2 in equation (24) must be zero for this particular problem: If the stock price ever reaches zero it will stay at zero, as the process (3) is lognormally distributed. In other words, 0 is an absorbing barrier for the stock price. This is not enough, however. Although the stock price might be zero, it is still mathematically possible to receive a dividend given a fundamental value V above zero. In this special case there are no dividends and the value of the resale option must therefore be zero if the stock price is zero. Then it follows that $H(0) = 0$ in equation (24), and since $erfi(0) = 0$, it must be that $A_2 = 0$.

Value matching

The option to 'cash in' on the mispricing translates into receiving the price of the stock while giving up the value of future dividends. Thus, the value of the resale option at exercise is $G(S^*, V^*) = S^* - V^*$.[10] Making the substitution $G(S, V) = V \cdot H(R)$, we get $V^* \cdot H(R^*) = S^* - V^*$ and therefore:

$$H(R^*) = R^* - 1. \tag{29}$$

Smooth pasting

We also require that the value matching condition is obtained 'smoothly' so that there is no discontinuity leading to the boundary condition. Equating the first derivatives of the option and the value at exercise, gives $\frac{\partial}{\partial S} G(S^*, V^*) = \frac{\partial}{\partial S} (S^* - V^*)$ and

$\frac{\partial}{\partial V} G(S^*, V^*) = \frac{\partial}{\partial V} (S^* - V^*)$. Using (14) and (16) to rewrite these requirements into the function H instead, we get for the left hand side:

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[10]Normally, exercise is denoted with the time T but as there is no time dimension in this problem exercise is denoted with *. When the stock price is stochastic, it makes no sense to postulate a rule saying for example that we should exercise on the 14th of February. It does make sense, however, to say that we should exercise when the stock is overpriced by 30 %.

$$\frac{\partial}{\partial S}G(S^*,V^*) = \frac{\partial}{\partial R}(V^* \cdot H(R^*)) \cdot \frac{\partial R}{S} = H_R(R^*)$$

$$\frac{\partial}{\partial V}G(S^*,V^*) = \frac{\partial}{\partial V}(V^*) \cdot H(R^*) + V^* \cdot H_R(R^*) \cdot \frac{\partial R}{\partial V} = H(R^*) - R^* \cdot H_R(R^*).$$

The smooth pasting requirements are hence:

$$H_R(R^*) = 1 \tag{30}$$

$$H(R^*) - R^* \cdot H_R(R^*) = -1. \tag{31}$$

The requirement in (31) follows directly from (29) and (30), and is consequently redundant.

Solution

Taken all together, the boundary conditions (29) and (30) give that:

$$H(R^*) = A_1 \sqrt{\frac{\pi a}{2\eta}} e^{-\frac{(a+b)^2}{2a\eta}} erfi\left(\frac{a+\eta \ln R^*+b}{\sqrt{2a\eta}}\right) = R^* - 1. \tag{32}$$

$$H_R(R^*) = A_1 e^{-\frac{(a+b)^2}{2a\eta}} \cdot e^{\left(\frac{a+\eta \ln R^*+b}{\sqrt{2a\eta}}\right)^2} \cdot \frac{1}{R^*} = 1. \tag{33}$$

Dividing 32) with (33) now gives:

$$\frac{\sqrt{\frac{\pi a}{2\eta}} erfi\left(\frac{a+\eta \ln R^*+b}{\sqrt{2a\eta}}\right)}{e^{\left(\frac{a+\eta \ln R^*+b}{\sqrt{2a\eta}}\right)^2} \cdot \frac{1}{R^*}} = R^* - 1 \tag{34}$$

Rewriting the solution to the optimal stopping problem hence is:

$$\sqrt{\frac{\pi a}{2\eta}} erfi\left(x^*\right) - \frac{(R^*-1)e^{x^{*2}}}{R^*} = 0, \text{ where } x^* = \frac{a+\eta \ln R^*+b}{\sqrt{2a\eta}} \tag{35}$$

The expression in (35) must be solved numerically for R^*, which can be done in excel. Having obtained the optimum in this way, the value of the resale option can be obtained by equation (33), giving:

$$A_1 = \frac{R^*}{e^{-\frac{(a+b)^2}{2a\eta}} \cdot e^{x^{*2}}}. \tag{36}$$

By simplifying $x = \dfrac{a + \eta \ln R + b}{\sqrt{2a\eta}}$ and using equations (13) and (24), we can write the value of the resale option as:

$$G(S,V) = V \cdot R^* \sqrt{\frac{\pi a}{2\eta}} e^{-x^{*2}} erfi\left(x\right). \tag{37}$$

Note that the set $\{R, x\}$ depends on the initial values of $\{S, V\}$, whereas $\{R^*, x^*\}$ refers to the optimal values of $\{S^*, V^*\}$. As $erfi\left(0\right) = 0$, the solution only makes economic sense when $x \geq 0$ (otherwise the value of the resale option is zero). The resale option is out-of-the-money when the observed share price is below the fundamental value. When it is sufficiently deep out-of-the-money, it will eventually become worthless. The value of the share then coincides with the fundamental value, which constitutes the lower limit.

3.2. On the assessment of the exercise date

We are also interested in the probability that the threshold of R^* is reached within a specified period of time. Equation B.3 in Appendix B, gives the process for the (logarithm of) mispricing ratio as:

$$d\ln R = -\eta \ln R(t)dt + \sigma_R dW_R \tag{38}$$

where:

$$\sigma_R = (\sigma_S^2 + \sigma_V^2 - 2\rho\sigma_S\sigma_V)^{1/2}$$

This process is mean reverting according to the Ornstein-Uhlenbeck process. As the observed price is reverting towards the fundamental value, also the mispricing ratio will be mean reverting. If the observed price is higher than the fundamental value, then $\ln R$ is positive and the drift is therefore negative. On the other hand, if the stock price is lower than the fundamental value, $\ln R$ is negative and the expected price move is upwards. As $\ln R(T)$ is normally distributed, the mispricing ratio $R(T)$ is lognormally distributed as illustrated in Figure 4.
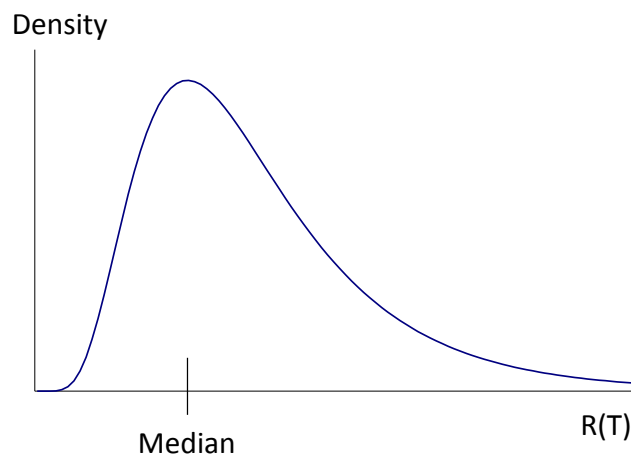

Figure 4. The distribution of R (ratio of observed share price to fundamental value).

The distribution in Figure 4 is not symmetric, implying that the size of positive bubbles can be much higher than the size of negative bubbles. This is a consequence of the limited liability charter, allowing only positive stock prices, but - in this modelling at least - it is true 'that markets can stay irrational a lot longer than you and I can remain solvent'.[11]

The median of a lognormal distribution is where the density is at the highest and asymptotically this is when $R=1$ and hence the stock price and the fundamental value

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[11]This quote is often attributed to Keynes, however there is to our knowledge no evidence that it really stems from him.

coincide. Hence, we do not assume any systematic overpricing. On the other hand, when overpricing does occur, it might be substantial. The right tail of a lognormal distribution can be quite long. This perhaps also makes it clear that it is potentially misleading to talk about an expected mispricing, 'an expected value of R'. It is therefore better to talk about the probability that R^* has been reached *before* a certain point in time, than the expected time until R^* is reached.

Unfortunately there is no analytical solution to the probability density function of the 2first hitting time2 for an Ornstein-Uhlenbeck process. Alili et al (2004) provides a series expansion and a 3-dim Bessel bridge solution, but none of these are trivial.[12] We have therefore resorted to Monte-Carlo simulation and equation (B.4) in Appendix B provides the solution to (38) as:

$$\ln R(T) = \ln R(t_0)e^{-\eta(T-t_0)} + \omega_T \sigma_R \varepsilon_R, \tag{39}$$

where $\omega_T = \sqrt{\dfrac{1 - e^{-2\eta(T-t_0)}}{2\eta}}$ and the random variable ε_R is standardised normal. This discrete time stochastics can be used to simulate the probability that R^* has been reached within a time period of one year.

It can be pointed to that both Lee et al (1999) and Lee and Lee (2013) empirically find that the ratio of price to value appears to be mean-reverting, as our model predicts. However, they attribute this to a time-varying expected return and some risk factor not captured in the Fama-French 3-factor model. These are alternative explanations and very difficult to distinguish from market mispricing as it will have the same time-series properties.

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[12]Yi (2010) shows that for a barrier of zero, the probability of hitting the barrier prior to T is double the probability that the barrier is breached at time T. This result is convenient for risk control, but not useful in our model setting.

4. A numerical example and economic consequences

Assume that the fundamental value $V = \$100$ and that the current share price also is $S = \$100$. The stock is presently thus correctly priced. However, one cannot be certain that the stock price equals the fundamental value also in the future. If the stock price unexpectedly drifts upwards – should the investor sell at a price of \$105, or perhaps wait until the price has reached \$150?

With the parameter setting in Figure 2 above, we assume no dividends and that the required rate of return $\mu = 8\%$. The speed of reversion towards the fundamental value is $\eta = 0.5$. The volatility of the fundamental value $\sigma_V = 20\%$, the volatility of mispricing $\sigma_S = 30\%$, and the two processes are assumed to be uncorrelated.

For these parameters, solving equation (35) numerically gives the critical ratio $R^* = 1.38$. This means that the observed price should be 38 % higher than the fundamental value before it is optimal to exercise the resale option and sell the stock. Given a fundamental value of \$100, S^* is thus \$138. The value of the resale option is given by equation (37) as \$9.8. Monte-Carlo simulation using the process (39), gives the probability of reaching the critical ratio within one year to be around 30 %.

There is no idea for an informed investor to sell the stock at a price below the fundamental value as this is what in expectation will be received in form of dividends if one is prepared to wait. It thus makes sense to depict the strike of the resale option as $\max\{S - V; 0\}$ where $V = \$100$ and show the value of the resale option $G(S,V)$ in relation to that. This is done in Figure 5, and the pattern is recognizable from an ordinary call option.
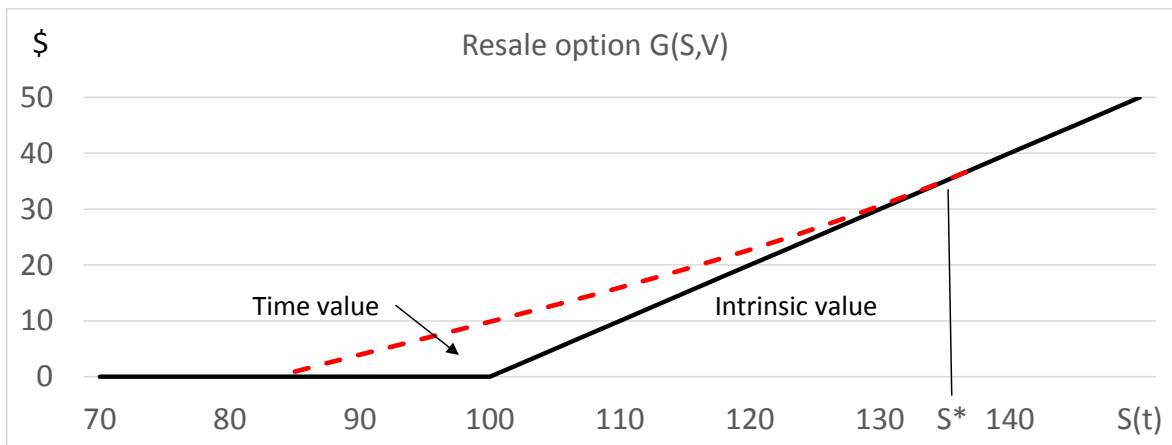
Figure 5. The value of the resale option resembles a call option, and can be divided into an intrinsic value and a time value.

Imagine next a situation where the fundamental value still is $100 but the stock price is $130. The optimal exercise of $S^* = \$138$ is unchanged as no fundamentals have changed, but we are much closer to exercise. In fact, the probability of hitting S^* within a year has increased to about 75 %. The value of the resale option is now $30.6.It is worth $30.6 to give up the present value of expected dividends worth $100 to be able to exercise at the optimum stock price of $138. However, it is possible to exercise immediately and receive $130, so the extra value of waiting is only $0.6. As for an ordinary call option, we can split the value of the resale option into an intrinsic value of $30 and a time value of $0.6.[13]In determining whether to exercise or holding on to the option it is only the time value of the option that is interesting, as that is what you give up by exercising. When both the fundamental value of the stock and the price is at $100, there is no intrinsic value of the resale option so the whole value of $9.8 is a time value. On the other hand, when the price has increased to $130, you only give up the time value of $0.6 when exercising. Clearly we

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[13] The total value of an option can be split into two parts. The intrinsic value is payment from an immediate exercise. However, the option value is higher than this, reflecting the fact that one does not have to exercise immediately - the time value. Option traders often think in terms intrinsic and time value as it provides more intuition than an option pricing formula.

cannot expect an informed investor to exercise at $100 or there around. It require a substantial mispricing to trigger exercise.

Next, we draw a surface plot of how the optimal exercise price S^* changes when the speed of reversion η and the irrational volatility σ_S changes.
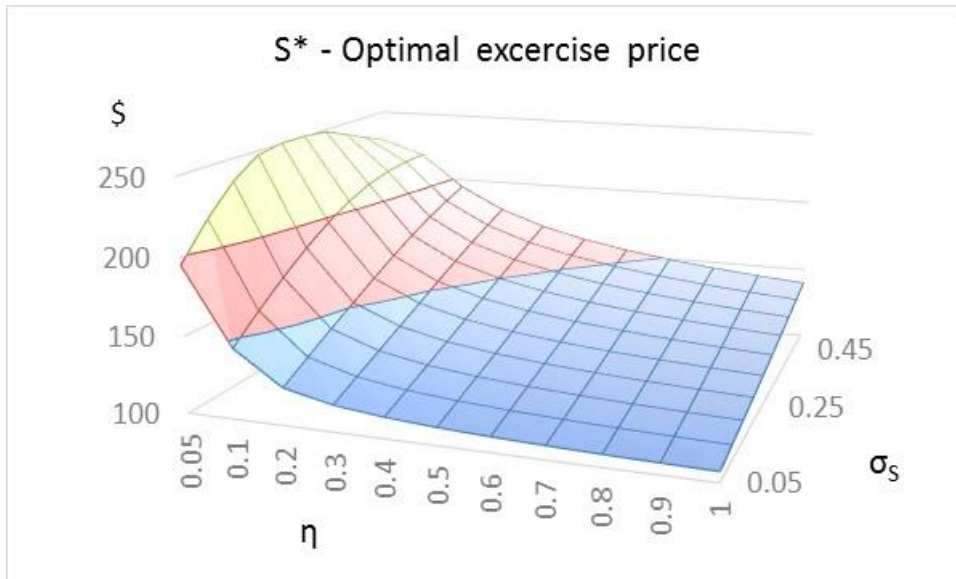

Figure 6. Optimal exercise price as a function of mispricing parameters.

The resale option has no value when the speed of reversion is high and the irrational volatility low. This is the situation in the lower right corner. The optimal exercise price shown in Figure 6, therefore approaches $100. When there is not much irrationality to start with and the speed at which mispricing is corrected is high, one should cash in on the mispricing as soon as possible as the market then quickly becomes efficient.

On the other hand, when irrational volatility is high and the speed of reversion is slow, it will be more valuable to wait rather than exercising as soon as mispricing occurs. The mispricing might get larger in the future. Setting $\eta = 0.05$ means that $e^{-0.05} = 95$ % of the mispricing remains after 1 year without any new information arriving. Combining this with

an irrational volatility of 30%, gives an optimal exercise price of $S^* = \$253$. The value of the resale option would then be $42 (not shown). In an irrational market, there is hence no incentive to exercise as soon as a mispricing occurs.

For an ordinary call option, the standard Black-Scholes model does not work particularly well when the time to maturity is very short. Assessed values then become very sensitive to the passage of time and the chosen volatility assumptions. This is not a model flaw, and in practice many investors exercise a few days before maturity as option prices get erratic. Something similar happens in our model when both σ_S and η becomes very small. There is hardly any irrationality in the market in the sense that the price all of a sudden deviates from the fundamental value as σ_S is small. On the other hand, when this happens there is no correction either as η is small. The model tends to give an optimal exercise price that is very high, or even goes to infinity.

Going back to the Google example in Figure 1 above, an accounting based fundamental value was simply estimated by a market-to-book multiple as earnings growth had been very stable. For this case, $\sigma_V = 4.5\%$ and using equation (39) $\eta = 1.28$ and $\sigma_S = 19.5\%$. Our calculations are based on quarterly data and the correlation between the fundamental value and the observed price is for simplicity set to 0. Equation (35) then gives the critical ratio $R^* = 1.10$ and the implication of this solution is illustrated in Figure 7 below.

Google

$R(T) = \dfrac{S(T)}{V(T)}$

$R^* = 1.10$

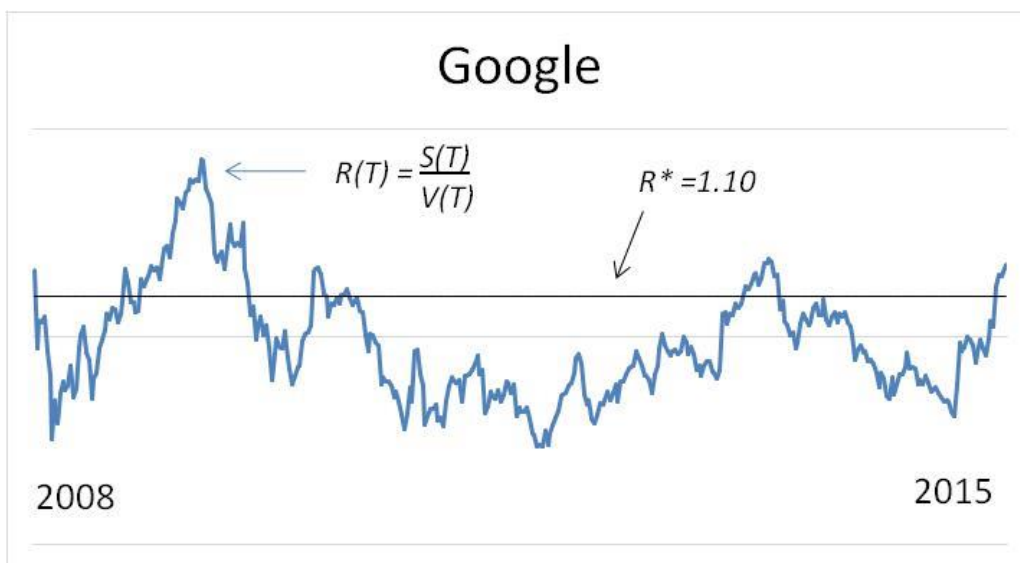2008   2015

Figure 7. The optimal selling price for the Google stock is when the ratio
between the observed share price and the fundamental value is 1.10.

When the stock price exceeds the fundamental value by 10 %, the stock should be sold. The possibility of an even larger mispricing then no longer outweighs the expectation that the mispricing will disappear.

A natural question at this point is whether there also exists an optimal purchase price, in the same way that the optimal selling price was just derived? Our answer to this question is negative, and we can use the analogy of an ordinary American call option to illustrate this point. It is well known from the literature that it can never be optimal to exercise an American call on a non-dividend paying stock prior to maturity (cf. Hull, 2009; chapter 9). However, when the stock is dividend paying, the valuation of an American call also becomes an optimal stopping problem. In the presence of one big dividend, it might be optimal to exercise the call just prior to that dividend and the theoretical value of the option is determined by this optimal strategy. If you could buy the American call cheaper than the theoretical value, it is a good buy. In the same way, if you can pick up the resale option cheaper than the theoretical value, it is also a good buy. The point is here that both the American call option and the resale option are optimal stopping problems of when to *exit the position*. We can neither for the American call, nor for the resale option, talk about an optimal buying strategy. In both cases, the optimal stopping problem has *no mirror image* of an optimal starting problem. The best strategy for the informed investor is to buy as many shares as possible up to the point where the stock price equals the optimal exercise price and then sell out. Presumably this might not work on a grand scale. However, it clearly works in our model setting as the stock price process has been assumed to be exogenously given.

By contrast, in a market equilibrium model, exercising at $138 would arguably affect the stock price, and knowing this will affect the stock price beforehand. The point of optimal

exercise will therefore depend on the resources of different investors clienteles. Cleary, such assumptions are difficult to handle in an equilibrium model. In our modelling, the fundamental investor is assumedly marginal and he/she does not affect the market price. It is hence a model over how a singular informed investor should behave given the existence of noise traders.

Nevertheless, our results are important for understanding how an equilibrium can be established. Our modelling indicates that a substantial mispricing is required before informed fundamental investors exercise their sale options. The consequence is hence that mispricing can be sustained for very long periods before being corrected. Frankel and Lee (1998) – in a test of the accounting based Residual Income Valuation model – writes that: "Our findings that prices converge to value estimates gradually over longer horizons (beyond 12-months) is puzzling." However, in light of our modelling of the resale option, this is rather what should be expected. Market prices do not easily correct themselves in a setting with more persistent noise traders.

A crucial ingredient in our modelling is of course the assumed information asymmetry between informed investors and noise traders. Without this asymmetry, the value of the resale option would be 0 and the only value of holding a share would be its fundamental value. Companies, the financial press, financial analysts and accounting regulators all have an important role in reducing the information asymmetry between market investors. As for companies and regulators, there is evidence that companies with higher accounting quality suffered smaller price declines during the stock market crash of 1929 (cf. Barton and Waymire, 2004; Leftwich, 2004). Regarding the Asian financial crisis in 1997 - 1998, Mitton (2002) found a robust association between accounting quality and stock performance. Callen et al (2013) furthermore has showed that low accounting quality is typically associated with a significant delay in price reactions in the U.S. stock market.

As regards the role of the business press in mitigating the information asymmetry between investors, Bushee et al (2010) and Drake et al (2014) have found that market mispricing decreases with more intense media coverage in the business press. Bushee and co-authors

claim that the bid-ask spread decreases and the trading volume increases with increasing press coverage. In Drake et al (2014) the positive correlation between next year's (abnormal) stock return and the cash flow for the current year, is found to be smaller when there is more press coverage through the Wall Street Journal Newswire. Interestingly, the effect is limited to news wires and there is no effect for full articles, suggesting that the business press is better at just conveying information than in processing the value effects of the information. Mohanram (2014) shows that the presence of analyst cash flow forecasts, as compared to only earnings forecasts, reduce the accruals anomaly.[14] Bhojraj et al (2009), on the other hand, attribute the reduction in the accruals anomaly to the introduction of the Sarbanes-Oxley Act and the FASB introducing stricter rules for restructuring charges. However, from the perspective of this paper, it does not really matter which motive that best fits the reduction in the accruals anomaly. The important point is that companies, analysts, media and regulators all are central in improving the information efficiency of the stock markets.

A potentially interesting application of our modelling of the share resale option value is the well-established discount on closed end investment companies. Managers of investment companies presumably possess superior information about their portfolio companies, and can hence be expected to not trade on the market price alone. However, if the management of a closed-end investment company (for whatever reason) is never expected to sell its portfolio companies, it has in essence waived the resale option value of its shares. Hence the investment company should trade at a lower market price than a corresponding investment company which is prepared to sell off its portfolio companies. Closed end investment companies that trade more aggressively should then be priced with a smaller discount, given that stock market investors believe that they can take advantage of the resale option. This is in line with previous empirical results in Boudreaux (1973) and Malkiel (1995), showing that higher portfolio turnover tends to be associated with smaller discounts for closed end investment companies or funds. However, whether the resale

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[14]This anomaly, originally discovered in Sloan (1996), is that a trading strategy based on taking long positions in companies with negative accruals and a short positions in companies with positive accruals, generate superior returns over longer holding periods.

option, costly arbitrage, or company realisation taxes best explain the discount of closed end investment companies still appear to be an unresolved issue.

Another potential application of our modelling of the resale option is the so called carry trading in currency exchange markets. Carry trading implies borrowing in a currency with low interest rates and investing in a currency with high interest rates. The chain of events normally starts with a country where the expected inflation is higher than in similar countries, and the nominal interest rates therefore have increased as compared to the other countries. This creates an inflow of investors ("noise traders") wanting to exploit the higher interest rates. Our resale option modelling would then suggest that informed investors should threat the currency rate as exogenously driven by irrational investors, and that the informed investors should not exit their position until the mispricing of the inflation struck currency has reached a certain threshold level.

5. Summary

We have analysed the potential pricing effect of the resale option of quoted company shares, as first suggested by Harrison and Kreps (1978). If share prices deviate from their fundamental values, fundamental investors who reliably can assess these values have an information advantage as compared to noise traders who only act on observed market prices. A fundamental investor has then the opportunity to sell the stock when it is overpriced, a resale option. We suggest a new pricing modelling approach where observed share prices mean revert towards the fundamental value in expectation, modelled as a geometric Brownian motion. The pricing process is lognormal and contains both a permanent and a temporary component. It also incorporates excess volatility in line with Shiller (1981), and a declining variance ratio in line with Cochrane (1988).

We value the resale option as a stopping problem using dynamic programming and derive a solution as to when the stock should be sold. An insight of this kind can provide fund managers with a tool to determine a rational selling strategy of their portfolio holdings. If fundamental investors have no immediate incentive (due to the need for liquidity) to trade

and drive the stock price towards the present value of expected future dividends, the mechanism for correcting market mispricing can be quite weak. The conclusion is therefore that stock market mispricing will be more persistent than the standard notion of the efficient market hypothesis predicts. Note that this is not a consequence of irrational informed investors, rather this follows from "rational behaviour in an irrational market". The difference between a fundamental investor and a noise trader is likely to at least partly be caused by information asymmetries among capital investors. Initiatives by companies, financial analysts and/or accounting regulators to reduce such information asymmetries are therefore important. Perhaps more important than previously realised, as stock market mispricing does not easily correct itself.

Our stock price modelling provides a potential explanation for the well-known discount on closed end investment companies. To the extent that managers of such companies do not trade their stock portfolio holdings, they have in principle waived the right to the resale option value of their shareholdings. Consequently such closed end investment companies should trade at a higher discount than investment companies that have chosen to trade their portfolio shares. Carry trading is another area of application. Although a currency might be mispriced in fundamental terms, investors might still be prepared to hold on to interest bearing cash holdings in weak currencies in order to (for some time) earn the difference in interest rates between weaker versus stronger currencies.

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Appendix A: Derivation of the stock price process

The fundamental value of a share follows a geometric Brownian motion, with fixed drift and diffusion coefficients $\mu - \delta$ and σ_V respectively, i.e.

$$dV(t) = (\mu - \delta)V(t)dt + \sigma_V V(t)dW_V(t). \tag{A.1}$$

With the time dependence suppressed, this is equation (1) in the main text. It is however easier to work with log values in the following derivations. Using Ito's lemma and letting lowercase v symbolise the logarithm, $v(t) = \ln V(t)$, gives:

$$dv(t) = \alpha dt + \sigma_V dW_V(t), \tag{A.2}$$

where

$$\alpha = \mu - \delta - \tfrac{1}{2}\sigma_V^2.$$

As $v(t)$ becomes a generalised Wiener process, the solution to (A.2) is:

$$v(T) = v(t_0) + \alpha(T - t_0) + \sigma_V \sqrt{T - t_0}\ \varepsilon_V \tag{A.3}$$

where ε_V is i.i.d. standard normal. The observed stock price is assumed to be mean reverting around the fundamental value according to the process

$$dS = \left[\tfrac{1}{2}\sigma_S^2 + \alpha + \eta[v(t) - \ln S(t,v)]\right]S(t)dt + \sigma_S S(t,v)dW_S(t),$$

where

$$Corr[W_V(t), W_S(t)] = \rho.$$

In the main text (A.4) is written as equation (3) with $v(t) = \ln V(t)$, and $\theta = \tfrac{1}{2}\sigma_S^2 + \mu - \delta - \tfrac{1}{2}\sigma_V^2$. We use the representation $\theta = \tfrac{1}{2}\sigma_S^2 + \alpha$ henceforth, as the first term soon will cancel out. The process for the observed stock price requires more work before it can be written in explicit form and not just as a differential. Once again, it is easier to work

with log values and we use lower case s to denote the log observed stock price. Defining $s(t, v) = \ln S(t, v)$ and using Ito's lemma implies with partial derivatives,

$$\frac{\partial s}{\partial t} = 0, \quad \frac{\partial s}{\partial v} = 0, \quad \frac{\partial s}{\partial S} = \frac{1}{S}, \quad \frac{\partial^2 s}{\partial S^2} = -\frac{1}{S^2},$$ that the log observed price is mean reverting

and follows an Ornstein-Uhlenbeck process,

$$ds = \left[\alpha + \eta[v(t) - s(t,v)]\right]dt + \sigma_S dW_S(t). \tag{A.4}$$

We want to derive the process of $s(t, v)$ subject to the stochasticity of v under the true probability measure. Applying Ito's lemma to the function $f(t, v, s) = se^{-\eta(T-t)}$, with the

partial derivatives: $\dfrac{\partial f}{\partial t} = \eta se^{-\eta(T-t)}, \quad \dfrac{\partial f}{\partial v} = 0, \quad \dfrac{\partial f}{\partial s} = e^{-\eta(T-t)}, \quad \dfrac{\partial^2 f}{\partial s^2} = 0,$ gives

$$\begin{aligned} df &= \{\eta s(t)e^{-\eta(T-t)} + [\alpha + \eta[v(t) - s(t,v)]]e^{-\eta(T-t)}\}dt + \sigma_S e^{-\eta(T-t)}dW_S(t) \\ &= \{\eta v(t)e^{-\eta(T-t)} + \alpha e^{-\eta(T-t)}\}dt + \sigma_S e^{-\eta(T-t)}dW_S(t). \end{aligned}$$

Integration of the notation above results in integral equation

$$\int_{t_0}^{T} df = \int_{t_0}^{T} \eta v(t)e^{-\eta(T-t)}dt + \int_{t_0}^{T} \alpha e^{-\eta(T-t)}dt + \int_{t_0}^{T} \sigma_S e^{-\eta(T-t)}dW_S(t).$$

Since f is defined as $f(t, v, s) = se^{-\eta(T-t)}$, the left hand side of the above equation becomes:

$$\int_{t_0}^{T} df = f(T) - f(t_0) = s(T) - s(t_0)e^{-\eta(T-t_0)},$$

In turn implying that

$$s(T) = s(t_0)e^{-\eta(T-t_0)} + \int_{t_0}^{T} \eta v(t)e^{-\eta(T-t)}dt + \int_{t_0}^{T} \alpha e^{-\eta(T-t)}dt + \int_{t_0}^{T} \sigma_S e^{-\eta(T-t)}dW_S(t). \tag{A.5}$$

The integral $\int_{t_0}^{T} \eta v(t) e^{-\eta(T-t)} dt$ needs further attention since $v(t)$ is a stochastic function. Using

Ito's lemma for the function $g(t,v) = v e^{-\eta(T-t)}$, gives with the partial derivatives

$$\frac{\partial g}{\partial t} = \eta v e^{-\eta(T-t)}, \quad \frac{\partial g}{\partial v} = e^{-\eta(T-t)}, \quad \frac{\partial^2 g}{\partial v^2} = 0,$$

and from equation (A.2), $dv(t) = \alpha dt + \sigma_V dW_V(t)$, the differential becomes:

$$dg = [\eta v(t) e^{-\eta(T-t)} + \alpha e^{-\eta(T-t)}] dt + \sigma_V e^{-\eta(T-t)} dW_V(t).$$

Integration now gives that

$$\int_{t_0}^{T} dg = \int_{t_0}^{T} \eta v(t) e^{-\eta(T-t)} dt + \int_{t_0}^{T} \alpha e^{-\eta(T-t)} dt + \int_{t_0}^{T} \sigma_V e^{-\eta(T-t)} dW_V(t),$$

which can be rewritten as:

$$\int_{t_0}^{T} \eta v(t) e^{-\eta(T-t)} dt + \int_{t_0}^{T} \alpha e^{-\eta(T-t)} dt = \int_{t_0}^{T} dg - \int_{t_0}^{T} \sigma_V e^{-\eta(T-t)} dW_V(t). \tag{A.6}$$

Substituting (A.6) into (A.5) gives

$$s(T) = s(t_0) e^{-\eta(T-t_0)} + \int_{t_0}^{T} dg - \int_{t_0}^{T} \sigma_V e^{-\eta(T-t)} dW_V(t) + \int_{t_0}^{T} \sigma_S e^{-\eta(T-t)} dW_S(t). \tag{A.7}$$

Furthermore, we have that

$$\int_{t_0}^{T} dg = g(T) - g(t_0)$$

$$= v(T) - v(t_0)e^{-\eta(T-t_0)} \tag{A.8}$$

$$= v(t_0) + \alpha(T - t_0) + \int_{t_0}^{T} \sigma_V dW_V(t) - v(t_0)e^{-\eta(T-t_0)},$$

and substituting (A.8) into (A.7) gives:

$$s(T) = s(t_0)e^{-\eta(T-t_0)} + v(t_0)(1 - e^{-\eta(T-t_0)}) + \alpha(T - t_0) +$$

$$+ \int_{t_0}^{T} \sigma_V(1 - e^{-\eta(T-t)})dW_V(t) + \int_{t_0}^{T} \sigma_S e^{-\eta(T-t)}dW_S(t). \tag{A.9}$$

The Ito integrals in equation (A.9) are next in line. Following Björk (1998; p. 43) it can be shown that for a deterministic function $h(t)$ and a process $Y(T)$ defined as

$$Y(T) = \int_{t_0}^{T} h(t)dW(t), \ Y(T) \text{ is normally distributed with zero mean and variance}$$

$$Var[Y(T)] = \int_{t_0}^{T} h^2(t)dt. \text{ Applying this observation makes it possible to rewrite } s(T) \text{ as:}$$

$$s(T) = s(t_0)e^{-\eta(T-t_0)} + v(t_0)(1 - e^{-\eta(T-t_0)}) + \alpha(T - t_0) +$$

$$+ [\sqrt{T - t_0} - \omega(T)]\sigma_V \varepsilon_V + \omega(T)\sigma_S \varepsilon_S, \tag{A.10}$$

where $\omega_T = \sqrt{\dfrac{1 - e^{-2\eta(T-t_0)}}{2\eta}}$, and the random variables ε_V and ε_S are uncorrelated and

standard normal. Equation (A.10) shows the stock price process in discrete time, in principle being a combination of a random walk and a mean reverting stochastic process. For a random walk, the variance increases linearly over time and this is captured by the factor $(T - t_0)$ in the second row of (A.10). The variance of a mean reverting process, on

the other hand, is asymptotically constant which is captured by the factor $\dfrac{1 - e^{-2\eta(T-t_0)}}{2\eta}$.

Altogether; the variance behaviour is described by Figure 2 in the main text. Also note that as

$$\lim_{\substack{\sigma_S \to 0 \\ \eta \to \infty}} s(T) = v(t_0) + \alpha(T - t_0) + [\sqrt{T - t_0}\,]\sigma_V \varepsilon_V, \tag{A.11}$$

the price process converges to the process for the fundamental value - equation (A.3) - if the 'irrational' volatility is low and the speed of reversion towards the fundamental value is high. In parameter estimations and Monte-Carlo simulations, the two standardised normal random variables ε_V and ε_S can be substituted by a single standardised normal variable, ε. Generally, for the parameters $\{a_V, a_S\}$ we have $a_V \varepsilon_V + a_S \varepsilon_S = [a_V^2 + a_S^2 + 2\rho a_V a_S]^{1/2} \varepsilon$, where $\rho = corr(\varepsilon_V, \varepsilon_S)$.[15] Applied to equation (A.10), this means that $s(T)$ can be written as expression (A.12) below, the base for our parameter estimations.

$$s(T) = c(T) + d(T)\varepsilon_T \tag{A.12}$$

where:

$$c(T) = s(t_0)e^{-\eta(T-t_0)} + v(t_0)(1 - e^{-\eta(T-t_0)}) + \alpha(T - t_0)$$

$$d(T) = [c_V^2(T) + c_S^2(T) + 2\rho c_V(T)c_S(T)]^{1/2}$$

where

$$s(t_0) = \ln S(t_0)$$

$$v(t_0) = \ln V(t_0)$$

$$\alpha = \mu - \delta - \tfrac{1}{2}\sigma_V^2$$

$$c_V(T) = [\sqrt{T - t_0} - \omega(T)]\sigma_V$$

$$c_S(T) = \omega(T)\sigma_S$$

$$\omega(T) = \sqrt{\frac{1 - e^{-2\eta(T-t_0)}}{2\eta}}$$

$$\rho = corr(\varepsilon_V, \varepsilon_S).$$

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[15]The most familiar application of this formula is perhaps the 'standard deviation of a portfolio with two assets'.

Appendix B: The process of the mispricing ratio *R*

The stopping problem of when an informed investor should sell the share is in the main text expressed in terms of the ratio R^*. Given the share price and fundamental value as of today, we now want to find the probability that this threshold is reached.

The definition of R is from equation (12)

$$R(S,V,t) = \frac{S(V,t)}{V(t)}, \tag{B.1}$$

and taking the logarithms we get:

$$\ln R = \ln \frac{S}{V} = s(v,t) - v(t). \tag{B.2}$$

Using the differentials in (A.2) and (A.4), the process for the logarithm can be written as

$$d\ln(R) = (\alpha + \eta[v(t) - s(t)] - \alpha)dt + \sigma_S dW_S(t) - \sigma_V dW_V(t).$$

The drift term in the above expression can be rewritten in terms of R using equation (B.2). The diffusion terms can be simplified by following the same steps as in equations (A.9) - (A.12). As Wiener processes are normally distributed with mean zero and a variance equal to the time increment, the two Wiener processes can be reduced to one. In all, these changes give the process for $\ln R(t)$ as

$$d\ln R = -\eta \ln R(t)dt + \sigma_R dW_R(t), \tag{B.3}$$

where

$$\sigma_R = (\sigma_S^2 + \sigma_V^2 - 2\rho\sigma_S\sigma_V)^{1/2}.$$

Equation (B.3) is an Ornstein-Uhlenbeck process as equation (A.4), although easier to solve. In comparison with equation (A.12), we get the solution as

$$\ln R(T) = \ln R(t_0)e^{-\eta(T-t_0)} + \omega_T \sigma_R \varepsilon_R, \tag{B.4}$$

where $\omega_T = \sqrt{\dfrac{1 - e^{-2\eta(T-t_0)}}{2\eta}}$ and the random variable ε_R is standardised normal. This is

equation (39) of the main text. The probability that $R(T)$ exceeds the critical ratio where exercise is triggered, is then:

$$\text{Prob}\!\left(R(T) \geq R^*\right) = 1 - N\!\left(\frac{\ln R^* - \ln R(t_0)e^{-\eta(T-t_0)}}{\omega_T \sigma_R}\right). \tag{B.5}$$

The interesting statistic is the probability that R^* has been reached *before* time T. This probability is higher than (B.5) and can be obtained through Monte-Carlo simulation based on equation (B.4).

Appendix C: Notations

$G(S, V, t)$ = value of resale option.

$H(R)$ = resale option as a function of the mispricing ratio R.

$R(t) = S(t, V)/V(t)$ = mispricing in relative terms.

R^* = optimal exercise ratio, the "critical ratio".

$S(V, t)$ = market price of share at time t.

$V(t)$ = fundamental value of share = present value of the expected future dividends.

$W_S(t)$ = Wiener process of the share price.

$W_V(t)$ = Wiener process of the fundamental value.

$a = \frac{1}{2}(\sigma_V^2 + \sigma_S^2 - 2\rho\sigma_V\sigma_S)$ = constant in the differential equation of $H(R)$.

$b = \frac{1}{2}(\sigma_S^2 - \sigma_V^2) - \delta$ = constant in the differential equation of $H(R)$.

$s(v, t) = \ln S(v, t)$ = logarithm of share price at time t.

$v(t) = \ln V(t)$ = logarithm of fundamental value at time t.

$\alpha = \mu - \delta - \frac{1}{2}\sigma_V^2$ = drift parameter of the logarithm of fundamental value.

δ = dividend yield.

η = speed of mean reversion of observed stock price.

$\theta = \frac{1}{2}\sigma_S^2 + \mu - \delta - \frac{1}{2}\sigma_V^2$ = parameter for share price process with asymptotical convergence of S to V.

ρ = correlation between Wiener processes.

σ_V = diffusion parameter = the standard deviation of fundamental value.

σ_S = diffusion parameter of observed stock price (partly driven by "irrational" volatility).